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Can Neutrinos be Degenerate in Mass?

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ABSTRACT

We reconsider the possibility that the masses of the three light neutrinos of the Standard Model might be almost degenerate and close to the present upper limits from Tritium β decay and cosmology. In such a scenario, the cancellations required by the latest upper limit on neutrinoless double-β decay enforce near-maximal mixing that may be compatible only with the vacuum-oscillation scenario for solar neutrinos. We argue that the mixing angles yielded by degenerate neutrino mass-matrix textures are not in general stable under small perturbations. We evaluate within the MSSM the generation-dependent one-loop renormalization of neutrino mass-matrix textures that yielded degenerate masses and large mixing at the tree level. We find that $m_{\nu_e} > m_{\nu_\mu} > m_{\nu_\tau}$ after renormalization, excluding MSW effects on solar neutrinos. We verify that bimaximal mixing is not stable, and show that the renormalized masses and mixing angles are not compatible with all the experimental constraints, even for $\tan\beta$ as low as unity. These results hold whether the neutrino masses are generated by a see-saw mechanism with heavy neutrinos weighing $\sim 10^{13}$ GeV or by non-renormalizable interactions at a scale $\sim 10^5$ GeV. We also comment on the corresponding renormalization effects in the minimal Standard Model, in which $m_{\nu_e} < m_{\nu_\mu} < m_{\nu_\tau}$. Although a solar MSW effect is now possible, the perturbed neutrino masses and mixings are still not compatible with atmospheric- and solar-neutrino data.

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1. Introduction

Observations of solar and atmospheric neutrinos provide strong indications that neutrinos may oscillate between eigenstates with different masses m_{ν_i} [1, 2]. The indications from solar neutrinos are for a difference in mass squared $\Delta m^2_{solar} \sim 10^{-4}$ [3] to 10^{-10} eV^2 [4], whereas the atmospheric neutrino data favour $\Delta m^2_{atmo} \sim 10^{-2}$ to 10^{-3} eV^2 [1]. As is well known, oscillation experiments are not able to set the overall scale of the neutrino masses. However, there are some upper limits on these: astrophysical and cosmological constraints on dark matter suggest that $\Sigma_i m_{\nu_i} <$ few eV [5], and experiments on the endpoint of the Tritium β-decay spectrum suggest that $m_{\nu_i} < 2.5$ eV [6] for any mass eigenstate with a substantial electron flavour component [7]. The question then arises whether there are any indirect arguments bearing on the possibility that the three light neutrino masses might be approximately degenerate [8]-[15] and close to these upper limits.

Extending previous arguments in [11] to include a new upper limit on neutrinoless double-β decay [16], we argue in the next section that, within the context of such degenerate neutrinos, this may already exclude the large-angle Mikheyev-Smirnov-Wolfenstein (MSW) [3] solution to the solar-neutrino problem. In this case, one would be forced into the vacuum-oscillation solution, and hence extreme mass degeneracy to one part in 10^{10}. In the case of the large-angle MSW solution, the degeneracy would need to be to one part in 10^4 or more.

We subsequently discuss general features of the one-loop renormalization of neutrino mass matrices, using as an example one specific degenerate mass-matrix texture that accommodates neutrinoless double-β decay via bimaximal mixing. We argue that this and other degenerate textures are generically unstable with respect to small perturbations, so that mixing does not remain bimaximal and the neutrinoless double-β constraint is no longer respected. We evaluate within the Minimal Supersymmetric extension of the Standard Model (MSSM) the one-loop renormalization-group corrections to mass degeneracy in scenarios where the masses are generated either by a see-saw mechanism [17] at some high scale M_N close to M_{GUT}, or by an effective operator at some low scale Λ close to M_{SUSY}. We find that these corrections are significant, even for relatively small values of $\tan\beta$, and lead to the ordering $m_{\nu_e} > m_{\nu_\mu} > m_{\nu_\tau}$. In the case that neutrino masses of order $\mathcal{O}(2)$ eV arise via the see-saw mechanism, these corrections indicate that degenerate neutrinos are not compatible with all constraints, even for $\tan\beta$ as low as 1. In particular, we note that the ordering of neutrino masses is *incompatible* with MSW solutions, because they require $m_{\nu_e} < m_{\nu_\mu}, m_{\nu_\tau}$. If neutrino masses arise from non-renormalizable interactions at a scale Λ as low as 10-100 m_{SUSY}, the effects are smaller. Still, even in this case, we cannot obtain the required degeneracy for the vacuum oscillations, and MSW oscillations cannot be obtained.

Finally, we discuss briefly renormalization effects in the minimal Standard Model. Their expected magnitude is similar to that in the MSSM in the low-$\tan\beta$ regime, since for a single Higgs field all the quark and charged-lepton mass hierarchies have to arise purely from the Yukawa couplings, and hence the τ coupling is small. However, in this case the sign of the

Yukawa renormalization effects is opposite, tending to increase the magnitudes of the entries in m_{eff} and leading to $m_{\nu_e} < m_{\nu_\mu}, m_{\nu_\tau}$, as required by the MSW mechanism. However, bimaximal mixing again cannot be maintained, so that this framework also appears incompatible with the constraint from neutrinoless double-β decay.

This analysis shows that schemes with degenerate neutrinos are very problematic, contrary to solutions with large neutrino hierarchies. Since the former may be obtained from non-Abelian symmetry structures [15] and the latter from Abelian ones [18, 19, 20], renormalization-group effects on the neutrino mass eigenvalues may be providing important information about the underlying flavour structure of the fundamental theory.

2. Neutrinoless Double-β Constraints on Degenerate Neutrinos

It was pointed out by Georgi and Glashow [11] that, if the neutrino masses are close to the Tritium and cosmological upper limit so that relic neutrinos contribute at least one percent of the critical density of the Universe, then the upper limits on neutrinoless double-β decay require the mixing angle for solar neutrino oscillations to be almost maximal. This is because the neutrinoless double-β decay limit constrains the ee component of the Majorana neutrino mass matrix in the charged-lepton flavour basis [11]:

$$m_{eff}^{ee} \equiv |m_1\, c_2^2 c_3^2 e^{i\phi} + m_2\, c_2^2 s_3^2 e^{i\phi'} + m_3\, s_2^2\, e^{i2\delta}| < B\,. \tag{1}$$

where c_i, s_i denote $\cos\theta_i, \sin\theta_i$ in the conventional 3×3 mixing parametrization

$$\begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix} = \begin{pmatrix} c_2c_3 & c_2s_3 & s_2e^{-i\delta}\\ -c_1s_3 - s_1s_2c_3e^{i\delta} & +c_1c_3 - s_1s_2s_3e^{i\delta} & s_1c_2\\ +s_1s_3 - c_1s_2c_3e^{i\delta} & -s_1c_3 - c_1s_2s_3e^{i\delta} & c_1c_2\end{pmatrix}\begin{pmatrix}\nu_1\\ \nu_2\\ \nu_3\end{pmatrix}, \tag{2}$$

where the diagonal matrix m_{eff}^{diag} is $diag(m_1e^{i\phi}, m_2e^{i\phi'}, m_3)$, ϕ and ϕ' are phases in the light Majorana mass matrix, and B is the experimental upper bound on m_{eff}^{ee}. In schemes with degenerate neutrinos, the differences between the mass eigenvalues m_i in (1) may be neglected [11]. Moreover, given the upper limits on atmospheric oscillations into electron neutrinos established by Chooz [21] and Super-Kamiokande [1] , we follow [11] and set $\theta_2 \approx 0$. Thus (1) may be simplified to the form:

$$|\cos^2\theta_3\, e^{i\phi} + \sin^2\theta_3\, e^{i\phi'}| < \frac{B}{\overline{m}} \tag{3}$$

where $\overline{m} \approx 2$ eV is the conjectured common mass scale of the (almost-)degenerate neutrinos.

At the time of [11], the best available upper limit was $B < 0.46$ eV, and the constraint (3) could be satisfied for $\phi + \phi' \simeq \pi$ and $|\cos 2\theta_3| < 0.23$, leading to $\sin^2 2\theta_3 > 0.95$. This was incompatible with the small-angle MSW solution for the solar neutrino data, but consistent with either the large-angle MSW solution or the vacuum-oscillation solution.

Recently, however, a new upper limit $B < 0.2$ eV has been given [16], from which we infer $|\cos 2\theta_3| < 0.1$ and hence

$$\sin^2 2\theta_3 > 0.99. \tag{4}$$

Such maximal mixing is favoured by the vacuum-oscillation solution, but is disfavoured in the large-angle MSW solution, because $\sin^2 2\theta_3 = 1$ would yield an energy-independent suppression of all solar neutrinos [22]. This disagrees with the Homestake data by at least three standard deviations, although the question persists whether the data could tolerate the lower limit in (4). Several global fits to the solar-neutrino data have been published, including one by the Super-Kamiokande collaboration [23] that takes into account its recent measurements of the day-night effect and yields $\sin^2 2\theta_3 < 0.99$ [3]. Other global fits often give smaller upper limits on $\sin^2 2\theta_3$ [23].

We infer, provisionally, that the large-angle MSW solution is now excluded by neutrinoless double-β decay if the neutrinos are near-degenerate, forcing us into the vacuum-oscillation solution, in which case the neutrino mass degeneracy must be at the level of one part in 10^{10}. We could, however, imagine possible ways to evade this conclusion. Perhaps a small but non-trivial admixture of $\nu_\mu - \nu_e$ atmospheric oscillations could soften the bound (4), and/or perhaps the solar-neutrino data could be stretched to accommodate it: $\sin^2 2\theta_3 = 1$ is excluded just at the 99.8% confidence level. Alternatively, the constraint (4) would be weakened for degenerate neutrinos weighing less than 2 eV. We comment later how our conclusions would be affected if the large-angle MSW solution could be tolerated, in which case the neutrino mass degeneracy need only be to one part in 10^4.

Using the experimental information then available, a specific effective neutrino mass texture was proposed in [11, 12]:

$$m_{eff} \propto \begin{pmatrix} 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{2} & -\frac{1}{2} \\ \frac{1}{\sqrt{2}} & -\frac{1}{2} & \frac{1}{2} \end{pmatrix} \tag{5}$$

in the flavour basis where charged-lepton masses are diagonal, leading to bimaximal mixing. Other neutrino mass textures might be considered, depending on the theoretical assumptions and other phenomenological choices made. In the following, we shall use (5) as an example, but frame our discussion in terms sufficiently general that it could be extended to other model textures.

Any such texture can only be regarded as a first approximation, that might be modified by higher-order effects. These could include the possible contributions of higher-dimensional non-renormalizable operators. The above discussion suggests that any such contributions should change the mass eigenstates by at most one part in 10^{10} (or 10^4), which is considerably more delicate than the expected hierarchy $m_{GUT}/m_P \approx 10^{-2}$. In the absence of any detailed

[3]If the day-night effect were not included, the upper limit would be $\sin^2 2\theta_3 < 0.95$: the magnitude of the day-night effect improves the quality of the fit in this large-angle MSW region [23].

theory of such contributions, one cannot say that this is necessarily a problem. However, global symmetries are not normally expected to be exact at the Planck scale, so the mass-degeneracy constraint is potentially powerful.

Moreover, the mixing angles in such degenerate mass-matrix models are inherently unstable when higher-order perturbations are switched on, as we discuss in more detail later.

3. Renormalization-Group Effects on Neutrino Mass Textures

Calculable and potentially significant breakings of the neutrino mass degeneracies are provided by renormalization-group effects. However, these depend on the specific neutrino model framework adopted. We consider here two possibilities: one is the conventional see-saw, with a singlet-neutrino mass scale $M_N \sim 10^{13}$ GeV [4], and the other is a model where the light Majorana neutrino masses are simply generated by a new non-renormalizable interaction, such as $\nu_L \nu_L H H$, at a mass scale $\Lambda \sim 10^5$ GeV, close to $m_{SUSY} = 10^3$ GeV. Below we give numerical results for both scenarios.

In the see-saw case, between the GUT scale and the scale M_N of the heavy Majorana neutrinos, there is an effect on the mixing angle due to the renormalization-group running of the Dirac neutrino coupling Y_N [24]:

$$\begin{aligned} 8\pi^2 \frac{d}{dt}(Y_N Y_N^\dagger) &= \{-\sum_i c_N^i g_i^2 + 3(Y_N Y_N^\dagger) + \mathrm{Tr}[3(Y_U Y_U^\dagger) + (Y_N Y_N^\dagger)]\}(Y_N Y_N^\dagger) \\ &\quad + \frac{1}{2}\{(Y_E Y_E^\dagger)(Y_N Y_N^\dagger) + (Y_N Y_N^\dagger)(Y_E Y_E^\dagger)\} \end{aligned} \tag{6}$$

Here and subsequently we work at the one-loop level, and denote the renormalization-group scale by $t \equiv \ln \mu$. In the MSSM, $c_N^i = (3/5, 3, 0)$, and we denote the Dirac couplings of other types of fermion F by Y_F. It is apparent from (6) that large Yukawa couplings have a bigger effect on m_{33}^D than on the rest of the mass-matrix elements, and tend in general to lower Y_N. This alters the structure of the Dirac mass matrix, in turn affecting the magnitudes of the mixing angles. These effects become more relevant in examples where cancellations between various entries may lead to amplified mixing in m_{eff}. However, we assume here that any neutrino-mass texture [11] is defined at the characteristic scale M_N of the see-saw mechanism. Therefore, in the current discussion we use this first part of the run only in order to define the initial conditions for the gauge and Yukawa couplings at M_N, but not to modify the neutrino-mass texture. Since the exact form of m_{eff} depends on the right-handed Majorana mass matrix, we simply assume that this has the form that is required in order to lead to a specific texture at M_N.

[4] We note that the heavy Majorana neutrino masses M_N need not be degenerate. On the contrary, flavour symmetries indicate that should have a structure determined by the flavour charges of the N fields, and of other singlet fields in a given model. However, we do not discuss explicitly here this structure, which could also in principle affect the amount of renormalization-group running.

Having set the initial conditions for g_i and λ_i at M_N, we note that Y_N decouples below the right-handed Majorana-mass scale, where the relevant running is that of the effective neutrino-mass operator [25, 26]:

$$8\pi^2 \frac{d}{dt} m_{eff} = \{-(\frac{3}{5}g_1^2 + 3g_2^2) + \mathrm{Tr}[3Y_U Y_U^\dagger]\} m_{eff} + \frac{1}{2}\{(Y_E Y_E^\dagger) m_{eff} + m_{eff}(Y_E Y_E^\dagger)^T\} \ , \quad (7)$$

This is the basic equation for the running of the various entries of the effective light-neutrino mass matrix. We continue to work in the basis where the charged-lepton mass matrix is diagonal, which is also the basis in which the neutrino-mass texture is specified.

In previous work [27], we discussed the running of the (23) mixing angle, but here we focus more on the evolution of the m_{eff} entries themselves, extending our previous discussion to encompass solutions to the solar neutrino problem. For this purpose, we use the following differential equations for individual elements of the effective neutrino-mass matrix:

$$\frac{1}{m_{eff}^{ij}} \frac{d}{dt} m_{eff}^{ij} \quad = \quad \frac{1}{8\pi^2} \left(-c_i g_i^2 + 3h_t^2 + \frac{1}{2}(h_i^2 + h_j^2) \right) \quad (8)$$

It is convenient for the subsequent discussion to define the integrals

$$I_g \quad = \quad exp[\frac{1}{8\pi^2} \int_{t_0}^{t} (-c_i g_i^2 dt)] \quad (9)$$

$$I_t \quad = \quad exp[\frac{1}{8\pi^2} \int_{t_0}^{t} h_t^2 dt] \quad (10)$$

$$I_{h_i} \quad = \quad exp[\frac{1}{8\pi^2} \int_{t_0}^{t} h_i^2 dt] \quad (11)$$

where in I_{h_i} and h_i the subindex i refers to the charged-lepton flavours e, μ and τ. Simple integration of (8) yields

$$\begin{aligned} \frac{m_{eff}^{ij}}{m_{eff,0}^{ij}} \quad &= \quad exp\left\{ \frac{1}{8\pi^2} \int_{t_0}^{t} \left(-c_i g_i^2 + 3h_t^2 + \frac{1}{2}(h_i^2 + h_j^2) \right) \right\} \\ &= \quad I_g \cdot I_t \cdot \sqrt{I_{h_i}} \cdot \sqrt{I_{h_j}} \end{aligned} \quad (12)$$

where the initial conditions are denoted by $m_{eff,0}^{ij}$. As we have already noted, these conditions are defined at M_N, the scale where the neutrino Dirac coupling h_N decouples from the renormalisation-group equations.

Using (12), we see that an initial texture $m_{eff,0}^{ij}$ at M_N is modified to become

$$m_{eff} = \begin{pmatrix} m_{eff,0}^{11}\ I_e & m_{eff,0}^{12}\ \sqrt{I_\mu}\ \sqrt{I_e} & m_{eff,0}^{13}\ \sqrt{I_e}\ \sqrt{I_\tau} \\ m_{eff,0}^{21}\ \sqrt{I_\mu}\ \sqrt{I_e} & m_{eff,0}^{22}\ I_\mu & m_{eff,0}^{23}\ \sqrt{I_\mu}\ \sqrt{I_\tau} \\ m_{eff,0}^{31}\ \sqrt{I_e}\ \sqrt{I_\tau} & m_{eff,0}^{32}\ \sqrt{I_\mu}\ \sqrt{I_\tau} & m_{eff,0}^{33}\ I_\tau \end{pmatrix}$$

$$= \begin{pmatrix} \sqrt{I_e} & 0 & 0 \\ 0 & \sqrt{I_\mu} & 0 \\ 0 & 0 & \sqrt{I_\tau} \end{pmatrix} \cdot \begin{pmatrix} m^{11}_{eff,0} & m^{12}_{eff,0} & m^{13}_{eff,0} \\ m^{21}_{eff,0} & m^{22}_{eff,0} & m^{23}_{eff,0} \\ m^{31}_{eff,0} & m^{32}_{eff,0} & m^{33}_{eff,0} \end{pmatrix} \cdot \begin{pmatrix} \sqrt{I_e} & 0 & 0 \\ 0 & \sqrt{I_\mu} & 0 \\ 0 & 0 & \sqrt{I_\tau} \end{pmatrix} \tag{13}$$

at m_{SUSY}. We evaluate subsequently the integrals $I_{e,\mu,\tau}$ appearing in this renormalization. However, we can already extract some important qualitative information from (13).

• We first note that because of the factorization in (13), although the individual masses and mixings get modified, any mass matrix which is singular with a vanishing determinant - leading to a zero mass eigenvalue - remains so at the one-loop level. However, one should expect modifications at the two-loop level, which might be an interesting mechanism for generating a non-trivial but large neutrino-mass hierarchy.

• The Yukawa renormalization factors I_i are less than unity, and lead to the mass ordering $m_{\nu_e} > m_{\nu_\mu} > m_{\nu_\tau}$, to the extent that such naive flavour identifications are possible.

• One would expect that for values of I_τ substantially different from unity - which occur for large $\tan\beta$ in particular [5] - the renormalization effects on the (23) sector would be especially significant. However, there can be important effects even in the first-generation sector. These can be significant for two reasons. One is that, in view of the neutrinoless double-β decay analysis given above, very small mass differences may be required for addressing the solar neutrino problem, so we should keep even small renormalization effects in mind. The other is that, when off-diagonal entries in $m^{ij}_{eff,0}$ are large as in the sample texture (5), the I_τ renormalization effects feed through into all differences in mass eigenvalues.

• In the case of the mixing angles, we recall that renormalization effects may either enhance or suppress the mixing. In particular, it has been noted in the case that $m^{22}_{eff,0} = m^{33}_{eff,0}$ and atmospheric-neutrino mixing is maximal somewhere above the electroweak scale, the maximal mixing may not survive down to low energies if the τ Yukawa coupling is large, at least for certain textures, depending on the magnitude of the (23) entries. To illustrate this, we will make some generic comments on the case of 2×2 mixing, and we will return to neutrino-mixing effects for the texture (5) in the next section.

In the case of simple 2×2 mixing, we see from

$$\sin^2 2\theta_{23} = \frac{4(m^{33}_{eff,0})^2}{(m^{33}_{eff,0} - m^{22}_{eff,0})^2 + 4(m^{33}_{eff,0})^2} \tag{14}$$

that the degeneracy between m^{22}_{eff} and m^{33}_{eff} becomes important only if m^{23}_{eff} is of the same order as $m^{33}_{eff} - m^{22}_{eff}$. It is known that large neutrino-mass hierarchies can be generated by two-generation textures of the following forms [28, 27] in the basis where the charged leptons

[5] Most flavour-symmetry models in the literature assume large $\tan\beta$.

are diagonal:

$$\left(\begin{array}{cc} x^2 & x \\ x & 1 \end{array}\right), \left(\begin{array}{cc} 1 & \pm 1 \\ \pm 1 & 1 \end{array}\right), \left(\begin{array}{cc} 1 & 1 \\ -1 & -1 \end{array}\right), \left(\begin{array}{cc} 1 & x \\ x & 1 \end{array}\right) \tag{15}$$

where the first solution has a large but non-maximal mixing in contrast with the others. For the first texture [20], where $m^{22}_{eff,0} < m^{33}_{eff,0}$, the renormalization-group effects on the mixing clearly are negligible, and the same is also true for the third texture, due to the signs of the entries [6]. In the case of the second texture, one expects mild changes despite the fact that $m^{22}_{eff,0} = m^{33}_{eff,0}$, because $m^{23}_{eff,0}$ is large. In the fourth texture [9], $m^{22}_{eff,0} = m^{33}_{eff,0}$, and $m^{23}_{eff,0}$ is small, which is exactly the type of solution that is very unstable under renormalization-group running. Finally, we note that solutions with small hierarchies and large mixing [10] of the type

$$\left(\begin{array}{cc} x & 1 \\ 1 & x' \end{array}\right) \tag{16}$$

are also expected to be stable under the renormalization group.

To complete this algebraic discussion of renormalization-group effects, we now comment on the second case of interest, in which the neutrino-mass texture is assumed to be generated by non-renormalizable interactions at some relatively low mass scale $\Lambda \sim 10^5$ GeV, such as $\nu_L \nu_L HH/\Lambda$. In this case, there are no neutrino Dirac couplings to be renormalized, so the renormalization-group running between M_{GUT} and Λ is the same as equations (7) to (13), with Yukawa couplings only for quarks and charged leptons. Below the scale Λ, m_{eff} runs in the same way as we discussed previously below M_N, but the range of scales over which the renormalization must be computed is greatly reduced.

4. Numerical Results

We now present some numbers for I_τ and I_μ, in order to exemplify renormalization effects on the textures. We take as illustrative initial conditions [7] $\alpha^{-1}_{GUT} = 25.64$, $M_{GUT} = 1.1 \cdot 10^{16}$ GeV and $m_{SUSY} = 1$ TeV. We also take $h_t = 3.0$, and choose h_b/h_τ such that an intermediate scale M_N is consistent with the observed pattern of fermion masses [8]. We use

[6] The renormalization of three-generation textures is more complicated, as we see in the next section.

[7] Although the runnings of the gauge couplings and of h_t factor out, they nevertheless affect the magnitude of h_τ and hence the exact value of I_τ that one derives.

[8] The choice of input parameters needed to reproduce exactly the observed fermion masses depends on $\tan\beta$, but incorporating this refinement is unnecessary for our purposes. We comment only that, for small $\tan\beta$, an intermediate scale M_N may be consistent with the values of m_b and m_τ measured at low energies, at the cost of a certain deviation from bottom–tau mass unification [24], which may be $\sim 10\%$ for $M_N \approx 10^{13}$ GeV. However, this may be corrected [19], if there is sufficient mixing in the charged-lepton sector. For completeness, we note that we use $h_N = 3.0$: this choice has a small impact on the initial conditions at the scale M_N.

h_τ	I_τ	I_μ	m_3	m_2	m_1
3.0	0.826	0.9955	0.866	-0.952	0.997
1.2	0.873	0.9981	0.903	-0.966	0.998
0.48	0.9497	0.9994	0.962	-0.987	0.9996
0.10	0.997	0.99997	0.9478	-0.9993	0.99998
0.013	0.99997	1.00000	0.99998	-0.99999	1.00000

Table 1: *Values of I_τ and I_μ, for $M_N = 10^{13}$ GeV and different choices of h_τ. Also tabulated are the three renormalized mass eigenvalues calculated from the sample texture (5).*

the physical μ and e masses to fix h_μ, h_e. Finally, we take $M_N = 10^{13}$ GeV as our default, mentioning later the effects with a different choice. The values of I_τ and I_μ that we find with these inputs are given in the first three columns of Table 1 and plotted in Fig. 1. These results may be used to estimate the effects on the neutrino eigenvalues, mixings and mass differences in the specific texture (5), as shown in the last three columns of Table 1 and in Fig. 2.

We see that the renormalization-group effects on the neutrino-mass eigenvalues are significant. Since they are larger for the second- and third-generation leptons, as already commented, we find the following ordering of the light neutrino masses for textures with degenerate eigenvalues at M_N: $m_{\nu_e} > m_{\nu_\mu} > m_{\nu_\tau}$, which is *incompatible* with MSW solutions to the solar-neutrino problem. It is apparent from Table 1 and Fig. 2 that the breaking of the neutrino-mass degeneracy in this model is unacceptable for any value of h_τ corresponding to $1 < \tan\beta < 58$.

We now discuss the renormalization of the neutrino mixing angles, using as a particular example the texture (5). Initially we make a generic discussion, based on analytic formulas, and then we illustrate the discussion using two numeric examples. We consider the following parametrization of a perturbation from the initial texture, motivated by the structure (13):

$$m'_{eff} \propto \begin{pmatrix} 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}(1+\frac{\epsilon}{2}) \\ \frac{1}{\sqrt{2}} & \frac{1}{2} & -\frac{1}{2}(1+\frac{\epsilon}{2}) \\ \frac{1}{\sqrt{2}}(1+\frac{\epsilon}{2}) & -\frac{1}{2}(1+\frac{\epsilon}{2}) & \frac{1}{2}(1+\epsilon) \end{pmatrix} \tag{17}$$

where ϵ is a small quantity, which might arise from renormalisation group running or from some other higher-order effects such as higher-dimensional non-renormalizable operators. This perturbation lifts the degeneracy of the eigenvalues, which are now given by

$$1, \quad -1-\frac{\epsilon}{4}, \quad 1+\frac{3\epsilon}{4}$$

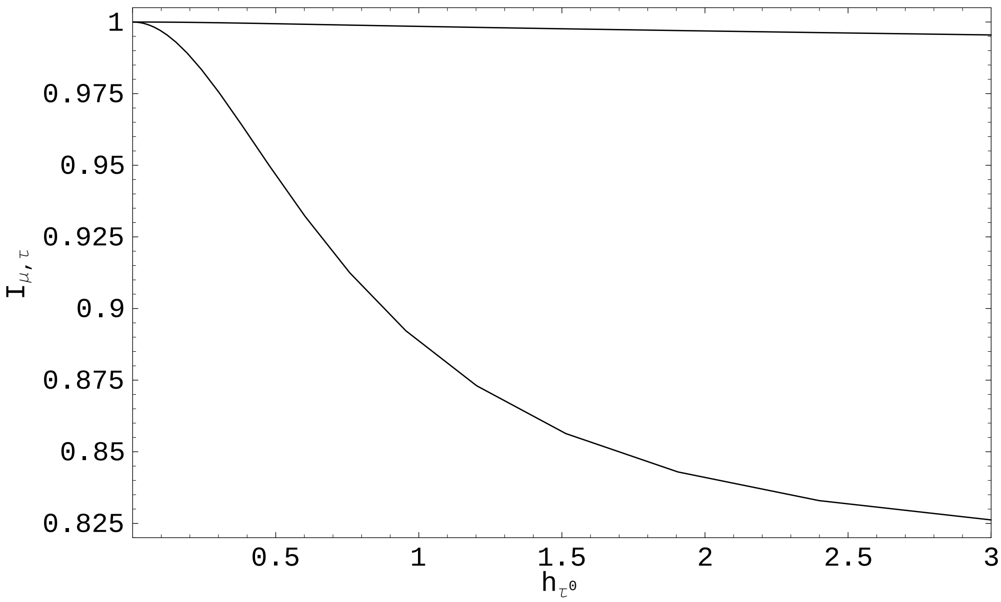

Figure 1: *Numerical values of I_μ and I_τ for different initial values of h_τ, assuming $M_N =$ 10^{13} GeV. The values of h^0_τ = 0.013, 0.03, 0.05 and 0.5 to 3 in steps of 0.5 correspond to tanβ 1, 3.8, 6.5, 43.8, 53.6, 56.3, 57.4, 57.9 and 58.2, respectively.*

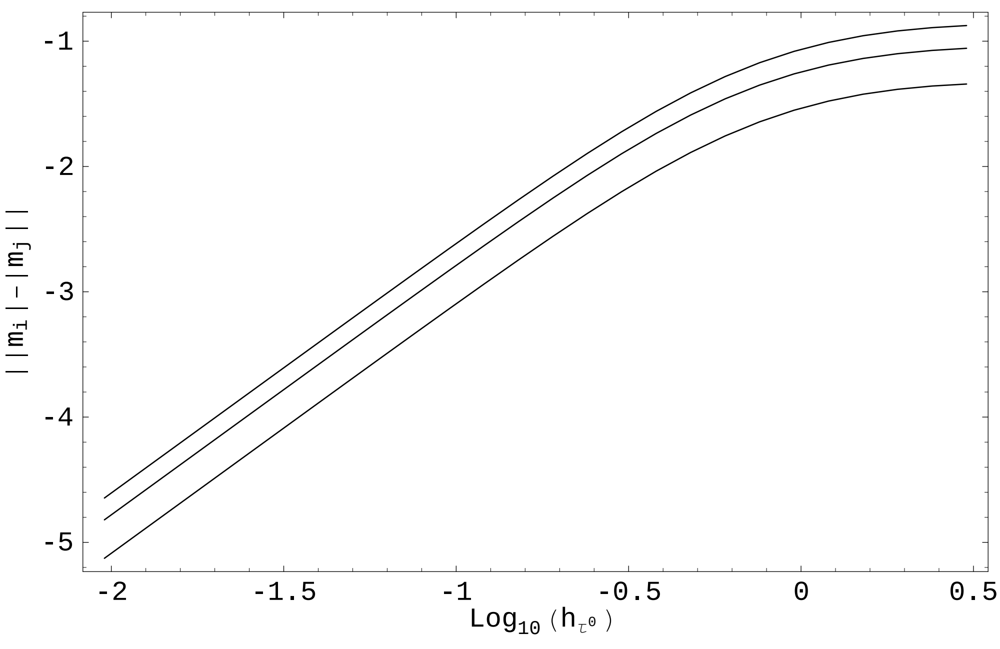

Figure 2: *Renormalization of m_{eff} eigenvalues for different initial values of h_τ corresponding to values of tanβ in the range 1 to 58, assuming the particular neutrino-mass texture (5) and $M_N = 10^{13}$ GeV. We see that the vacuum-oscillation scenario is never accommodated.*

To this order, the eigenvectors are independent of ϵ and given by

$$V_1 = \begin{pmatrix} \frac{1}{\sqrt{3}} \\ \sqrt{\frac{2}{3}} \\ 0 \end{pmatrix}, \quad V_2 = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{2} \\ -\frac{1}{2} \end{pmatrix}, \quad V_3 = \begin{pmatrix} \frac{1}{\sqrt{6}} \\ -\frac{1}{2\sqrt{3}} \\ \frac{\sqrt{3}}{2} \end{pmatrix} \tag{18}$$

so that the mixing expected in this type of texture does not depend on ϵ, as long as it is non-zero.

However, this mixing is *not* bimaximal. The vectors (18) are also eigenvactors of the unrenormalised texture (5). Since this unperturbed texture has two exactly degenerate eigenvalues, there is arbitrariness in the choice of eigenvectors: the vectors corresponding to the two degenerate eigenvalues can be rotated to different linear combinations, which are still eigenvectors of the neutrino mass matrix and still obey the orthogonality conditions. One example is the choice

$$\begin{aligned} V_1 &= \frac{1}{\sqrt{3}} V_1' + \sqrt{\frac{2}{3}} V_3' \\ V_3 &= \frac{1}{\sqrt{3}} V_3' - \sqrt{\frac{2}{3}} V_1' \end{aligned}$$

which gives

$$V_1' = \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{pmatrix}, \quad V_2' = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{2} \\ -\frac{1}{2} \end{pmatrix}, \quad V_3' = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{2} \\ \frac{1}{2} \end{pmatrix} \tag{19}$$

corresponding to bimaximal mixing: $\phi_1 = \frac{\pi}{4}$, $\phi_2 = 0$ and $\phi_3 = \frac{\pi}{4}$. However, one cannot in general expect this combination of eigenvectors to be stable when the degenerate texture is perturbed, and the above analysis shows that, indeed, it is not. On the contrary, it is the direction given by (18) that is stable, and the absence of the parameter ϵ in the eigenvectors indicates that we may expect only minor modifications in the mixing, for τ couplings in the range $3.0 - 0.013$.

We illustrate the instability of bimaximal mixing and the stability of the eigenvectors (18) with a numerical analysis of two extreme cases with $h_\tau^0 = 3$ and 0.013. Using the values of $I_{\tau,\mu}$ given in Table 1, we determine the full renormalized mass matrices to be:

$$m_{eff}^{1,ren} = \begin{pmatrix} 0 & 0.705 & 0.64 \\ 0.705 & 0.497 & -0.45 \\ 0.64 & -0.45 & 0.41 \end{pmatrix}, \quad m_{eff}^{2,ren} = \begin{pmatrix} 0 & 0.7071 & 0.7071 \\ 0.7071 & 0.5 & -0.499992 \\ 0.7071 & -0.499992 & 0.499985 \end{pmatrix}, \tag{20}$$

respectively, which are to be compared with the initial form (5) of the texture. Then, for

$$V_1 = \begin{pmatrix} 0.5804 & 0.8143 & 0.0065 \\ -0.7075 & 0.5003 & 0.4992 \\ 0.4032 & -0.2943 & 0.8665 \end{pmatrix}, \; V_2 = \begin{pmatrix} -0.57864 & -0.815578 & -0.00274 \\ 0.7071 & -0.5 & -0.5 \\ 0.406418 & -0.29126 & 0.866021 \end{pmatrix} \tag{21}$$

we find that

$$m^{1,ren}_{eff,diag} = V_1 \cdot m^{1,ren}_{eff} V_1^T = \begin{pmatrix} 0.997 & 0 & 0 \\ 0 & -0.952 & 0 \\ 0 & 0 & 0.866 \end{pmatrix} \tag{22}$$

and

$$m^{2,ren}_{eff,diag} = V_2 \cdot m^{2,ren}_{eff} V_2^T = \begin{pmatrix} 1.00000 & 0 & 0 \\ 0 & -0.99999 & 0 \\ 0 & 0 & 0.99998 \end{pmatrix} \tag{23}$$

reflecting the reverse mass ordering mentioned above. On the other hand, as we have already remarked, the eigenvectors (and thus the mixing matrices) are stable. It is easy to check that the matrices $V_{1,2}$ are unitary ones, and correspond to the values

$$\phi_1 \approx 0.52, \quad \phi_2 \approx 0, \quad \phi_3 \approx 0.95 \tag{24}$$

In our notation, atmospheric-neutrino mixing is controlled by the parameter ϕ_1, for which we find $\sin^2 2\phi_1 \approx 0.75$, whereas solar-neutrino mixing is controlled by ϕ_3, for which we find $\sin^2 2\phi_3 \approx 0.90$. We see therefore that even small perturbations of exact neutrino degeneracy cause large effects on the neutrino mixing angles, which then *conflict with the bounds from neutrinoless double-β decay*. This example shows that the mixing differs significantly from that postulated in the unperturbed degenerate texture, an effect not visible in a naive 2×2 analysis.

Up to now we have been discussing the situation where light neutrino masses arise through the see-saw mechanism, and therefore m_{eff} arises at a scale 10^{13} GeV. However, if m_{eff} arises at a significantly lower scale Λ, for example via effective operators of the form $\nu_L \nu_L HH/\Lambda$, as discussed at the end of the previous section, the integrals I_τ and I_μ are now much closer to unity. This happens because (i) the range where m_{eff} runs is significantly decreased, and (ii) the starting value of h_τ at Λ is also smaller, due to the run from M_{GUT} to Λ being over a relatively wide range. Shown in Table 2 and Fig. 3 are our calculations of I_τ and I_μ, using the same parameters as in Table 1 and Fig. 1, except that now we run down from $\Lambda = 10^5$ GeV [9]. The effects on the eigenvalues appear in Table 2 and Fig. 4, from which we see that, although the mass ratios are now closer to unity than in the previous case, the effects of the running can still not be neglected, when compared to the small mass differences required by the solar neutrino data. We again find that the full range $1 < \tan\beta < 58$ is excluded. On the other hand, the effect on the mixing angle is similar to the previous case, since it is practically unchanged under small perturbations, as we discussed earlier.

[9]Since the logarithmic range of renormalization-group running is short in this case, finite renormalization effects may be relatively more significant than in the $M_N = 10^{13}$ GeV case. However, their evaluation requires detailed modelling of thresholds, which lies beyond the scope of this paper. We consider it unlikely that the qualitative conclusions of this paper would be affected by their inclusion.

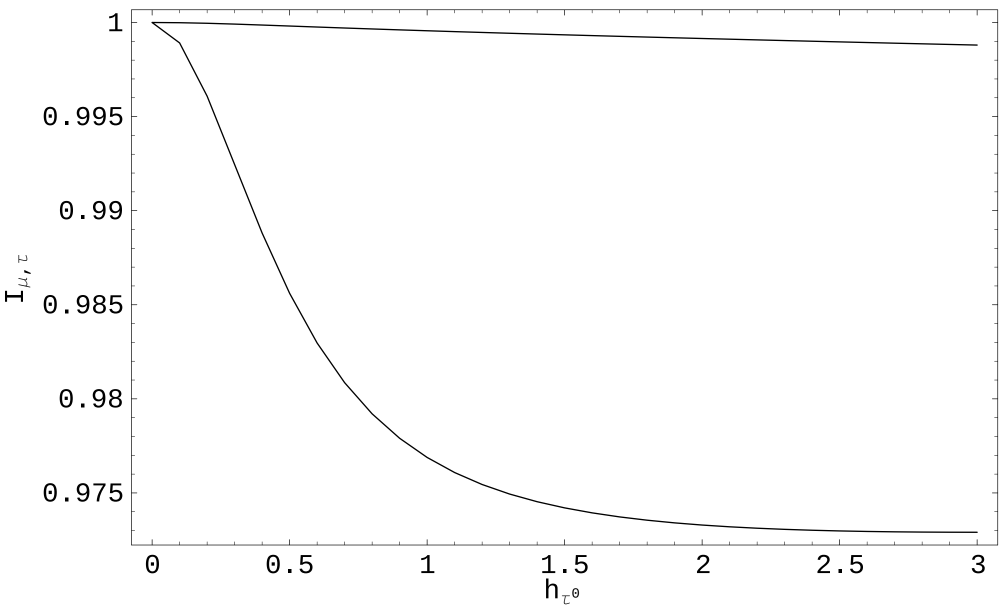

Figure 3: *Numerical values of I_μ and I_τ for different initial values of h_τ, assuming* $\Lambda = 10^5$ *GeV. The corresponding values of tanβ are roughly the same as in the previous case.*

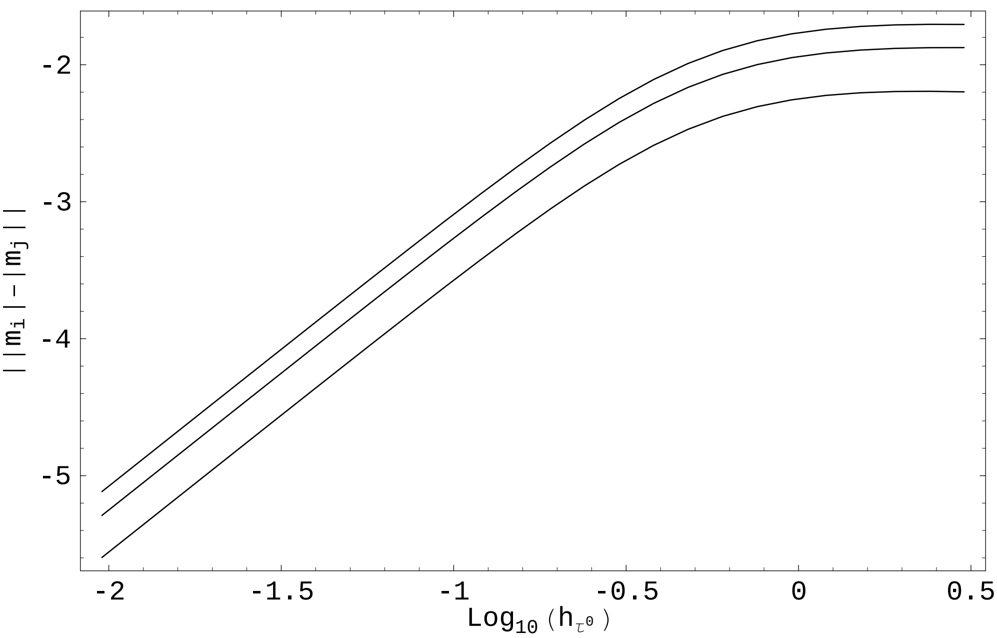

Figure 4: *Renormalization of m_{eff} eigenvalues for different initial values of h_τ corresponding to values of tanβ in the range 1 to 58, assuming the particular neutrino-mass texture (5) and* $\Lambda = 10^5$ *GeV. We see again that the vacuum-oscillation scenario is never accommodated.*

h_τ	I_τ	I_μ	m_3	m_2	m_1
3.0	0.973	0.9988	0.9795	-0.9929	0.9992
1.2	0.975	0.9995	0.9815	-0.9937	0.9996
0.48	0.986	0.9998	0.9897	-0.9965	0.9999
0.10	0.9999	0.99999	0.9993	-0.9997	0.99999
0.013	0.99999	1.000000	0.999992	-0.999997	1.00000

Table 2: *Values of I_τ and I_μ, for $\Lambda = 10^5$ GeV and different choices of h_τ. Also tabulated are the three renormalized mass eigenvalues calculated from the sample texture (5).*

We comment finally on the renormalization-group effects in the minimal non-supersymmetric Standard Model. The evolution equation for m_{eff} is

$$16\pi^2 \frac{dm_{eff}}{dt} = (-3g_2^2 + 2\lambda + 2S)m_{eff} - \frac{1}{2}((m_{eff}(Y_e^\dagger Y_e) + (Y_e^\dagger Y_e)^T m_{eff}), \tag{25}$$

where λ is the Higgs coupling: $M_H^2 = \lambda v^2$, and $S \equiv Tr(3Y_u^\dagger Y_u + 3Y_d^\dagger Y_d + Y_e^\dagger Y_e)$ [25]. Although the running of m_{eff} differs from the MSSM, the structure is similar. In particular, the contributions proportional to g_2, λ and S are the same for all entries, and thus the exponential factors that are obtained by integrating the renormalization-group equations multiply all entries, just as I_t and I_g did in the case of the MSSM. The term that affects the relative runnings of the various entries is again $\frac{1}{2}((m_{eff}(Y_e^\dagger Y_e) + (Y_e^\dagger Y_e)^T m_{eff}$, though with a sign opposite from the MSSM, meaning that now the Yukawa couplings increase the entries in m_{eff}. An important feature in the Standard Model, since it has only one Higgs field, is that the mass hierarchies between fermions with opposite electroweak hypercharge have to arise purely from the Yukawa couplings. Hence the starting value of the τ coupling is small in this case, and therefore the effects are expected quantitatively to be similar to those in the low-tan β MSSM, but in the opposite direction. This is interesting, since whereas starting from degenerate-mass neutrinos in the MSSM we expect low-energy neutrino hierarchies of the type $m_{\nu_\tau} < m_{\nu_\mu} < m_{\nu_e}$, in the Standard Model we expect the opposite ordering of masses: $m_{\nu_\tau} > m_{\nu_\mu} > m_{\nu_e}$, which is the right sign for MSW solutions of the solar-neutrino problem. However, the SM case shares with the MSSM case the instability in the bimaximal mixing. This means that the renormalized mass matrix is again incompatible with the constraint from neutrinoless double-β decay, even though the breaking of the mass degeneracy might appear compatible with the MSW solution to the solar-neutrino problem.

5. Conclusions

We have studied in this paper the circumstances under which neutrino masses can be degenerate and close to the present upper bounds from Tritium β decay and astrophysics.

We find that such schemes are severely constrained. In particular, the new upper limit on neutrinoless double-β decay [16], in combination with the rest of the solar-neutrino data, seems to exclude even the large-angle MSW solution to the solar-neutrino problem, and thus degenerate neutrinos may be compatible only with vacuum oscillations. However, in this case extreme mass degeneracy to one part in 10^{10} is required.

Even if such a degeneracy is guaranteed by a symmetry at a certain scale, we find that renormalization group effects lift this degeneracy. In the MSSM with light neutrino masses arising through the see-saw mechanism, the effects on the eigenvalues are larger for larger $\tan\beta$, and have the wrong sign for MSW solutions. For a given $\tan\beta$, the effects are reduced if m_{eff} arises via a non-renormalizable operator such as $\nu_L \nu_L HH/\Lambda$ at a significantly lower scale than the 10^{13} GeV required by the see-saw. Even in this case, however, the effects may not be neglected, in view of the extreme mass degeneracy that is required. Moreover, we find that even small perturbations shift the neutrino mixing angles by finite amounts, violating the constraints from neutrinoless double-β decay. Finally, we find in the minimal Standard Model renormalization effects that are qualitatively similar to those of the low-$\tan\beta$ MSSM, but with opposite signs, thus leading to reversed low-energy neutrino-mass ordering. In this case, the large-angle MSW solution may survive, but the instability in the degenerate neutrino mixing angles means that the neutrinoless double-β decay constraint is still violated.

Our analysis indicates that degenerate neutrino-mass textures have many problems when renormalization effects are taken into account. These results may provide hints on the appropriate framework for flavour symmetries, with Abelian models [18, 19, 20] apparently favoured.

References

[1] Y. Fukuda et al., Super-Kamiokande collaboration, Phys. Lett. B433 (1998) 9; Phys. Lett. B436 (1998) 33; Phys. Rev. Lett. 81 (1998) 1562.

[2] S. Hatakeyama et al., Kamiokande collaboration, Phys. Rev. Lett. 81 (1998) 2016; M. Ambrosio et al., MACRO collaboration, Phys. Lett. B434 (1998) 451.

[3] S. Nussinov, Phys. Lett. B63 (1976) 201; L. Wolfenstein, Phys. Rev. D17 (1978) 2369; S.P. Mikheyev and A.Y. Smirnov, Yad. Fiz. 42 (1985) 1441; Sov. J. Nucl. Phys. 42 (1985) 913.

[4] See, for example: J. N. Bahcall and W.C. Haxton, Phys.Rev. D40 (1989) 931; X. Shi, D. N. Schramm and J. N. Bahcall, Phys. Rev. Lett. 69 (1992) 717; P. I. Krastev and S. Petcov, Phys. Lett. B299 (1993) 94; N. Hata and P. Langacker, Phys. Rev. D50 (1994) 632 and references therein; N. Hata and P. Langacker, Phys. Rev. D52 (1995) 420.

[5] R.A.C. Croft, W. Hu and R. Davé, astro-ph/9903335, and references therein.

[6] V. Lobashev, Pontecorvo Prize lecture at the JINR, Dubna, January 1999, updating: A.I. Belesev et al., Phys. Lett. B350 (1995) 263.

[7] V. Barger, J.T. Weiler and K. Whisnant, Phys. Lett. B442 (1998) 255.

[8] C. D. Carone and M. Sher, Phys. Lett. B420 (1998) 83; A. S. Joshipura, Z. Phys. C64 (1994) 31; A. Ioannisian and J.W.F. Valle, Phys. Lett. B332 (1994) 93; F. Vissani, hep-ph/9708483.

[9] D. Caldwell and R. N. Mohapatra, Phys. Rev. D48 (1993) 3259;

[10] G.K. Leontaris, S. Lola, C. Scheich and J.D. Vergados, Phys. Rev. D 53 (1996) 6381; S. Lola and J.D. Vergados, Progr. Part. Nucl. Phys. 40 (1998) 71; B.C. Allanach, hep-ph/9806294.

[11] H. Georgi and S.L. Glashow, hep-ph/9808293.

[12] V. Barger et al., Phys. Lett. B437 (1998) 107; R. N. Mohapatra and S. Nussinov, Phys. Lett. B441 (1998) 299 and hep-ph/9809415; C. Jarlskog, M. Matsuda and S. Skadhauge, hep-ph/9812282; Y. Nomura and T. Yanagida, Phys. Rev. D59 (1999) 017303; S.K. Kang and C.S. Kim, Phys. Rev. D59 (1999) 091302.

[13] Solutions with degenerate neutrinos are included among the textures studied by G. Altarelli and F. Feruglio, Phys. Lett. B439 (1998) 112 and JHEP 9811 (1998) 021.

[14] J.P. Bowes and R.R. Volkas, J. Phys. G24 (1998) 1249; S. Davidson and S. King, Phys. Lett. B445 (1998) 191; A.J. Baltz, A.S. Goldhaber and M. Goldhaber, Phys. Rev. Lett. 81 (1998) 5730; H. Fritzsch and Z. Xing, Phys. Lett. B440 (1998) 313 and hep-ph/9903499; E. Ma, hep-ph/9812344 and hep-ph/9902392; M. Tanimoto, Phys. Rev. D59 (1998) 017304; N. Haba, Phys. Rev. D59 (1999) 035011; C. Giunti, Phys. Rev. D59 (1999) 077301; E.M. Lipmanov, hep-ph/9901316; T. Ohlsson and H. Snellman, hep-ph/9903252; A.H. Guth, L. Randall and M. Serna, hep-ph/9903464.

[15] Y. L. Wu, hep-ph/9810491, 9901245, 9901320; C. Wetterich, hep-ph/9812426; R. Barbieri, L. J. Hall, G. L. Kane, and G. G. Ross, hep-ph/9901228.

[16] L. Baudis et al., hep-ex/9902014.

[17] M. Gell-Mann, P. Ramond and R. Slansky, proceedings of the Supergravity Stony Brook Workshop, New York, 1979, eds. P. Van Nieuwenhuizen and D. Freedman (North-Holland, Amsterdam); T. Yanagida, proceedings of the Workshop on Unified Theories and Baryon Number in the Universe, Tsukuba, Japan 1979 (edited by A. Sawada and A. Sugamoto, KEK Report No. 79-18, Tsukuba).

[18] Some of the many references are: C. Wetterich, Nucl. Phys. B261 (1985) 461; G.K. Leontaris and D.V. Nanopoulos, Phys. Lett. B212 (1988) 327; Y.Achiman and T. Greiner, Phys. Lett. B329 (1994) 33; H. Dreiner et al., Nucl. Phys. B436 (1995) 461. Y. Grossman and Y. Nir, Nucl. Phys. B448 (1995) 30; P. Binétruy, S. Lavignac and P. Ramond, Nucl. Phys. B477 (1996) 353; P. Binétruy et al, Nucl. Phys. B496 (1997) 3, G. Altarelli and F. Feruglio, hep-ph/9812475.

[19] G.K. Leontaris, S. Lola and G.G. Ross, Nucl. Phys. B454 (1995) 25; S. Lola, hep-ph/9903203.

[20] S. Lola and G.G. Ross, hep-ph/9902283.

[21] M. Apollonio et al., Chooz collaboration, Phys. Lett. B420 (1998) 397.

[22] See, for example, C. Giunti in [14].

[23] Y. Fukuda et al., Super-Kamiokande Collaboration, Phys. Rev. Lett 82 (1999) 1810; S. Petcov and Y. Totsuka, private communications.

[24] F. Vissani and A. Yu. Smirnov, Phys. Lett. B341 (1994) 173; A. Brignole, H. Murayama and R. Rattazzi, Phys. Lett. B335 (1994) 345.

[25] K. Babu, C. N. Leung and J. Pantaleone, Phys. Lett. B319 (1993) 191.

[26] P.H. Chankowski and Z. Pluciennik, Phys. Lett. B316 (1993) 312; M. Tanimoto, Phys. Lett. B360 (1995) 41.

[27] J. Ellis, G.K. Leontaris, S. Lola and D.V. Nanopoulos, hep-ph/9808251, to appear in Eur. J. Phys. C; and J. Ellis, S. Lola and D.V. Nanopoulos, hep-ph/9902364.

[28] See, for example: A. Joshipura and A. Smirnov, Phys. Lett. B439 (1998) 103; S.F. King, Phys. Lett. B439 (1998) 350; G. Lazarides and N. Vlachos, Phys. Lett. B441 (1998) 46; M. Jezabek and Y. Sumino, Phys. Lett. B440(1998) 327; V. Barger, T. Weiler and K. Whisnant, Phys. Lett. B440 (1998) 1; R. Barbieri et al., JHEP 9812 (1998) 01; R. Barbieri, L.J. Hall and A. Strumia, Phys. Lett. B445 (1999) 407; E. Ma, Phys. Lett. B442 (1998) 238; Q. Shafi and Z. Tavartkiladze, hep-ph/9811282; F. Vissani, JHEP 9811 (1998) 025; C.H. Albright, K.S. Babu and S.M. Barr, Phys. Rev. Lett. 81 (1998) 1167; J.K. Elwood, N. Irges and P. Ramond, Phys. Rev. Lett. 81 (1998) 5064; M. Fukugita, M. Taminoto and T. Yanagida, hep-ph/9809554; W. Buchmüller and T. Yanagida, hep-ph/9810308; Y. Grossman, Y. Nir and Y. Shadmi, JHEP 9810 (1998) 007; C.D. Froggatt, M. Gibson and H.B. Nielsen, hep-ph/9811265; Z. Berezhiani and A. Rossi, hep-ph/9811447; K.S. Babu, J.C. Pati and F. Wilczek, hep-ph/9812538.